Using MMICAD as a Powerful General Purpose Mathematical Solver

Example 2: Curve Fitting of Data Points by the Least Squares Method

This application note describes how MMICAD can "curve fit" measurement data points. The approximating functions can be polynomial, exponential, logarithmic, power, or any desired mathematical expression. The measured data points are represented by g(x) and the fitting function is represented by f(x); the objective is to minimize the least square error defined as:

$$E = \sum | f(x_i) - g(x_i) |^2$$

MMICAD can optimize a condition proportional to E by defining the following residue equation:

$$Residue = | f(x_i) - g(x_i) |^2$$

In the **OPT**imization block, the residue equation is defined so as to attain a zero value with a weight W. For n data points, the built-in error function becomes:

$$Error\ Function = \frac{100}{n} \sum W * RESIDUE$$

When the optimization completes, the fitting function coefficients are obtained. Examples of fitting functions are:

$$Fit = \sum (A_n X_n) \quad n \geq 0 \quad (Polynomial)$$

$$Fit = e^{A_0 X} + A_1 \quad (Exponential)$$

$$Fit = A_0 LN(X) + A_1 \quad (Logarithmic)$$

$$Fit = X A_0 + A_1 \quad (Power)$$

EXAMPLE:

The technique can be illustrated for a common application, for example, obtaining 4th order polynomial fit for the capacitance versus voltage C(V) dependence of a diode. The MMICAD analysis file is shown in Figure 1. The procedure used to compose this file is listed below:

Step #1: FILE Block

The Capacitance Vs. Voltage data has to be in a data file format compatible with MMICAD. In this case, the data has to be arranged in a one-port format which requires three columns of information. In the example, the data file CV1.S1P has been pasted between the **FREEZE ON** (Start Comments) and **FREEZE OFF** (End Comments) block.

The data specification line in the example:

GHZ S RI R=50

informs MMICAD that the data file contains frequency data, real part of S11, and Imaginary part of S11 respectively.

To make our C(V) information a valid one port data file, the voltage and capacitance information are placed in the second and third column respectively, and the first column is arranged to contain the absolute values of the voltage column. This way, MMICAD treats the voltage information as real part of S11, and the capacitance information is treated as the imaginary part of S11. The "dummy" frequency column contains the voltage progression. The user must ensure that this column contains no duplicate points, all of which are greater than or equal to zero.

Step #2: VAR Block

The coefficients of the polynomials to be determined are defined using standard syntax as shown in Figure 1.

Step #3: PROC Block

The independent variable of the fit function is assigned to the voltage (real part of S11), while the FIT function is compared to the measured data, or the capacitance (imaginary part of S11) in the least square function definition LSQ.

Step #4: FREQ Block

This example contains no frequency dependence. However, the frequency list here is arranged to correspond one to one with the voltage column. This ensures that the fit covers all the voltage points.

Step #5: OPT Block

The least squares function is defined here to attain zero value with a weight of one.

Step #6: OUT Block

In this example, two output frames are displayed. One is the comparison of the measured and fitted points. Figure 2 shows this with the final optimized coefficients listed at the end of the analysis file in Figure 1. The second output frame shows the plot of the least square error as a function of voltage, and cumulatve error of the fit (by integrating the least square error). In this example, the cumulative error obtained at the last data point after the optimization is complete is less than 0.0002 using the Taurus Optimization. The final polynomial fit function is:

$$pF) = 0.217 + 0.0575V + 0.0740V^2 + 0.0362V^3 + 0.00475V^4 \qquad -3.5 \leq V$$

```
FILE NAME: DATAFIT.CKT

Least Squares Data Points Fitting using MMICAD

FILES
 \MMICAD\WORKING\CV.S1P DATAFILE 37 1P FREQ   ! One Port File Listing Shown Below

VAR
  A0=? -1e+008 1 1e+008 ?                              ! Final Optimized Results Shown Below
  A1=? -1e+008 1 1e+008 ?
  A2=? -1e+008 1 1e+008 ?
  A3=? -1e+008 1 1e+008 ?
  A4=? -1e+008 1 1e+008 ?

PROC
 DATA=DATAFILE IM[S11]                                 ! Dependent Variable= Capacitance
 X=DATAFILE RE[S11]                                    ! Independent Variable= Voltage
 FIT= A0+ A1*X+ A2*X^2+ A3*X^3+ A4*X^4                 ! 4th order Polynomial Fit Function C(V)
 LSQ=(FIT-DATA)^2                                      ! Least Square Error

FREQ
 SWEEP 0 3.5 0.1                                       ! Sweeps the Count column in data file

OPT
 OUTVAR LSQ EQ 0   W=1                                 ! Optimize for zero Least square error

OUT
 OUTVAR RE[DATA] Function                              ! Output of Data points Vs. Count
 OUTVAR RE[FIT] Function                               ! Output of Polynomial Function Vs. Count
 OUTVAR INTGR[LSQ] Residue                             ! Integrated Least Square Error
 OUTVAR RE[LSQ] Residue                                ! Least Square Error

GRID
 RANGE 0 3.5 0.5
 FUNCTION 0 0.25 0.05

LABEL
MMICAD Least Squares Data Fitting

FREEZE ON                                              ! Start of the Comments block

*********** Data File Listing ************
! Capacitance Vs. Voltage data
# GHZ S RI R=50
! Count  Voltage  Capacitance(pF)
  0.0    -0.0    .2119012
  0.1    -0.1    .2088151
  0.2    -0.2    .206014
  0.3    -0.3    .2045544
  0.4    -0.4    .2040018
  0.5    -0.5    .2040399
  0.6    -0.6    .2043893
  0.7    -0.7    .2047432
  0.8    -0.8    .2052289
  0.9    -0.9    .2052954
  1.0    -1      .2050246
  1.1    -1.1    .2042244
  1.2    -1.2    .203184
  1.3    -1.3    .2015105
  1.4    -1.4    .1994608
  1.5    -1.5    .1972855
  1.6    -1.6    .1946204
  1.7    -1.7    .1915958
  1.8    -1.8    .18839
  1.9    -1.9    .1849746
  2.0    -2      .1812159
  2.1    -2.1    .1770579
  2.2    -2.2    .1728088
  2.3    -2.3    .1680606
  2.4    -2.4    .1627043
  2.5    -2.5    .156969
  2.6    -2.6    .1508393
  2.7    -2.7    .1440026
  2.8    -2.8    .1361717
  2.9    -2.9    .1278431
  3.0    -3      .1185317
  3.1    -3.1    .1088934
  3.2    -3.2    .1000723
  3.3    -3.3    9.226853E-02
  3.4    -3.4    8.624505E-02
  3.5    -3.5    8.137859E-02

*********** RESULTS ************
Initial --> Final Error Function = 159995.7605 --> 0.0006

 A0=? -1e+008 0.217088 1e+008 ?
 A1=? -1e+008 0.0574594 1e+008 ?
 A2=? -1e+008 0.0739672 1e+008 ?
 A3=? -1e+008 0.0362175 1e+008 ?
 A4=? -1e+008 0.00474697 1e+008 ?

FIT=0.217088 +0.0574594*X +0.0739672*X^2 +0.0362175*X^3 +0.00474697*X^4

Remember... Capacitance in pF and -3.5  ≤ X ≤ 0

FREEZE OFF                                             ! End of the comments block
```

Figure 1 MMICAD Analysis for the Capacitance vs. Voltage Fit

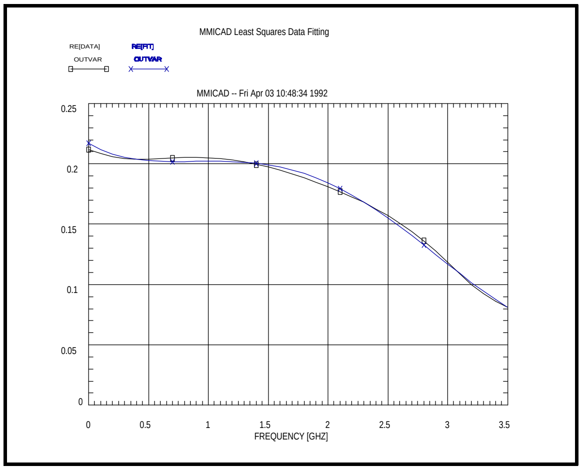

Figure 2 Fitted vs. Measured Data Points

CUSTOMIZING THE NEW MMICAD MODEL:

The derived Functional fit can now be used in MMICAD as a capacitance within the CAP component model by explicitely writing the equation (CAP n1 n2 C=..) whenever it is referenced. There are, however, advantages in writing a customized stand-alone model for the diode. Primarily, the diode model can be stored for use at any time using the **INCLUDE** statement. The voltage dependent capacitor one port model is assigned the name VCAP and is used within VCAP.CKT circuit file shown in Figure 3. Within the model description, the equation defining the **LOC**al **VAR**iable FIT is multiplied by 1e-12 to give FARADs for dimensions, and is also divided by **DIMCAP**. **DIMCAP** is assigned whatever value given for capacitance dimension in the **DIM** statment. The incorporation of **DIMCAP** into the model insures that the capacitance value is always correct for any user-specified dimension.

In Figure 3, MMICAD is set to **PARAM** sweep by the **MODE** command, and the diode voltage is assigned to **PARAM**eter while the frequency is held at 10 GHZ. This allows the user to sweep the capacitance as a function of voltage. The output of the sweep is shown in the SMITH chart in Figure 4.

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FILE NAME: VCAP.CKT

VCAP=Voltage Dependent Capacitance Model Implemented with Least Squares Data Points
Fitting Using MMICAD.
4th order Polynomial Fit, Range of Model -3.5  ≤ X ≤ 0

MODE PARAM                  ! Sweep MMICAD in PARAMeter Mode

GLOBAL
  DIM  FREQ=1e+009  RES=1  COND=0.001  CAP=1e-012  IND=1e-009  LNG=1e-00 6
TIME=1e-012

CKT                         ! VCAP Model with proper dimensioning
 MODVAR X=-0.1
 LOCVAR FIT={(0.217088+0.0574594*X+0.0739672*X^2+0.0362175*X^3&
            +0.00474697*X^4)*1E-12/DIMCAP}
 CAP 1 0 C=FIT
 DEF1P 1 VCAP ( X )

CKT                         ! VCAP Referenced here
 VCAP 1 2 X=PARAM           ! Assigns Voltage=PARAM
 DEF2P 1 2 VCAPMDL

FREQ
 FIXED 10                   ! Use this frequency while sweeping PARAMeter
 SWEEP 1 10 .1

PARAM                       ! Voltage is swept here
 SWEEP 0 -3.5 -0.1

OUT
 VCAPMDL SMI[S11] SMITH
```

Figure 3 Listing of VCAP.CKT Circuit File

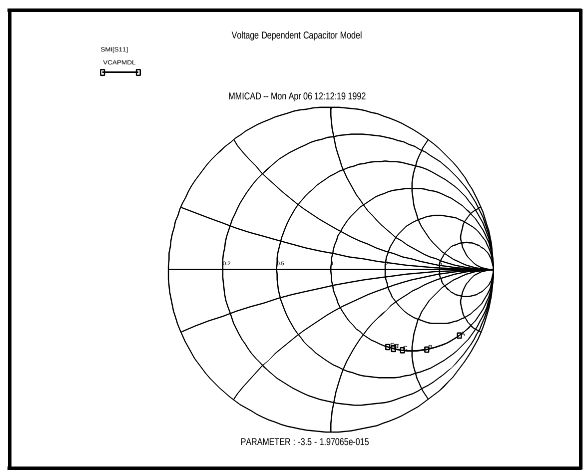

Figure 4 Capacitance vs. Voltage of the Diode