Using MMICAD as a Powerful General Purpose Mathematical Solver

Example 1: Solving Complex Nonlinear Equations

Complex equations are often encountered in microwave design, for example when solving for wave propogation in transmission media, matching and filter network synthesis, noise calculations, etc. With MMICAD's comprehensive mathematical functions library, in-line processing block, and three built-in optimizers, microwave designers can now use the program as a powerful complex nonlinear equation solver.

GENERAL SYNTAX for n * m equations:

$$B_1 = A_{11} * F(Vars_{11}) \odot A_{12} * F(Vars_{12}) \odot \ldots A_{1n} * F(Vars_{1n})$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$B_m = A_{m1} * F(Vars_{m1}) \odot A_{m2} * F(Vars_{m2}) \odot \ldots A_{mn} * F(Vars_{mn})$$

where:

- the A and B coefficients are real or complex numbers of the form a+jb
- $\odot$ represents either of the following operations: + - * / ^
- F(Vars) are functions of one or more variables which can be any of the mathematical functions listed in table 1.

ABS	Absolute Value
ACOS	Arc Cosine
ACOSH	Arc Hyperbolic Cosine
ANG	Angle (180 to +180)
ASIN	Arc Sine
ASINH	Arc Hyperbolic Sine
ATAN	Arc Tangent
ATANH	Arc Hyperbolic Tangent
CNJ	Complex Conjugate
COS	Cosine
COSH	Hyperbolic Cosine
EXP	Exponential Base e
FRAC	Fractional Part
IM	Imaginary Part
J0	Bessel Function of the First Kind-Order 0
J1	Bessel Function of the First Kind-Order 1
KKP	Elliptic Integral
LN	Logarithm Base e
LOG10	Logarithm Base 10
MAG	Magnitude
PHA	Angle (0 to 360)
RAD	Angle (Radians)
RE	Real Part
RND	Random Number
SIN	Sine
SINH	Hyperbolic Sine
TAN	Tangent
TANH	Hyperbolic Tangent
Y0	Bessel Function of the Second Kind-Order 0
Y1	Bessel Function of the second Kind-Order 1

Table 1 List of MMICAD Mathematical Functions

MMICAD ANALYSIS FILE:

The implementation of the n * m equations in MMICAD requires the construction of an analysis file. There are a few simple steps:

Step #1: Configure the VAR block

The arguments of the expressions are the unknowns to be determined, and they are placed in the VARiable block in standard optimization variable syntax, for example:

X=? 0 0.5 1 ?

Step #2: Configure the PROC block

The entire right hand side of each equation is assigned to a variable. For example:

EQN1= 5*X^2/EXP(Y)

Here, EQN1 is an OUTVAR variable.

STEP #3: OPT Block Syntax

Each equation is set to the real or imaginary part of the B coefficient:

OUTVAR RE[EQN1] EQ B_{re}
OUTVAR IM[EQN1] EQ B_{im}

where $B=B_{re}+J*B_{im}$. In the above example, the following equation:

3+J*5 = 5*X^2/EXP(Y)

would take the following Optimization statment:

OUTVAR RE[EQN1] EQ 3
OUTVAR IM[EQN2] EQ 5

Step #4: Configure the FREQ block

In most circumstances, there is no frequency dependence. However, at least one arbitrary frequency point must be swept using the STEP command.

Step #5: Configure the OUT block

A user-specified output block must be included. One could, for example, define residue equations where the right hand side is subtracted from the left hand side of each equation.

EXAMPLE:

Given the following (3 * 3) equations where X and Y are real
variables and Z is a complex variable.

$$-0.072 + j0.27 = -0.33X^2 - 0.51Y^3 + 0.42Z^2$$

$$-0.17 + j0.40 = 0.64\ln X + \frac{0.87e^{Y}}{Z^2}$$

$$-0.025 - j0.05 = \frac{0.85\sqrt{X}Y}{Z^2}$$

It is known that the solutions lie between 0 and +1. To solve these equations, the MMICAD analysis file shown in Figure 1 is developed according to the steps above. The frame RESIDUE can be chosen to monitor the progress. The output results are arranged in a form of residue of the equations superimposed on a polar chart. Figure 2 shows the residues on a polar chart prior to optimization, and Figure 3 shows the residues after optimization is complete. The optimized results are pasted between the FREEZE ON (start comments) and FREEZE OFF (end comments).

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FILE NAME: COMPEQN.CKT
NOTES:     General Nonlinear Complex Equations Solver
--------------------------------------------------------------------------------------------------------------
VAR
                  ! Final Optimized Values Listed Below
 X=? 0 0.2 1 ?                  ! X is a real variable
 Y=? 0 0.2 1 ?                  ! Y is a real variable
 Z1=? 0 0.2 1 ?
 Z2=? 0 0.2 1 ?
 Z={ Z1+J*Z2}                   ! Z is a complex variable

PROC
 EQN1= -0.33*X^2-0.51*Y^3+0.42*Z^2                ! Equation #1
 EQN2= +0.64*LN(X)+0.87*EXP(Y)/COSH(Z)            ! Equation #2
 EQN3= +0.85*SQRT(X)*Y/Z/Z                        ! Equation #3
!
 ERR1=EQN1-(-0.072+J*0.27)       ! Residue for Equation #1 for display
 ERR2=EQN2-(-0.17-J*0.40)        ! Residue for Equation #2 for display
 ERR3=EQN3-(-0.025-J*0.15)       ! Residue for Equation #3 for display

FREQ
 STEP 1

OPT
 OUTVAR RE[EQN1] EQ -0.072 W=1   ! Set Real part of Eqn #1 to -0.072
 OUTVAR IM[EQN1] EQ 0.27 W=1     !Set Imaginary part of Eqn #1 to 0.27
 OUTVAR RE[EQN2] EQ -0.17 W=1          ! Set Real part of Eqn #2 to -0.17
 OUTVAR IM[EQN2] EQ -0.40 W=1          ! Set Imaginary part of Eqn #2 to -0.40
 OUTVAR RE[EQN3] EQ -0.025 W=1   ! Set Real part of Eqn #3 to -0.025
 OUTVAR IM[EQN3] EQ -0.15 W=1          ! Set Imaginary part of Eqn #3 to -0.15

OUT
 OUTVAR POL[ERR1] RESIDUE        ! Output frame for Error display
 OUTVAR POL[ERR2] RESIDUE
 OUTVAR POL[ERR3] RESIDUE

LABEL
MMICAD Nonlinear Complex Equations Solver

FREEZE ON
************** Final Results **************
Initial --> Final Error Function = 167.5155 --> 0.4442

 X=? 0 0.12291 1 ?
 Y=? 0 0.333524 1 ?
 Z1=? 0 0.522184 1 ?
 Z2=? 0 0.618774 1 ?

FREEZE OFF
```

Figure 1 MMICAD Example File

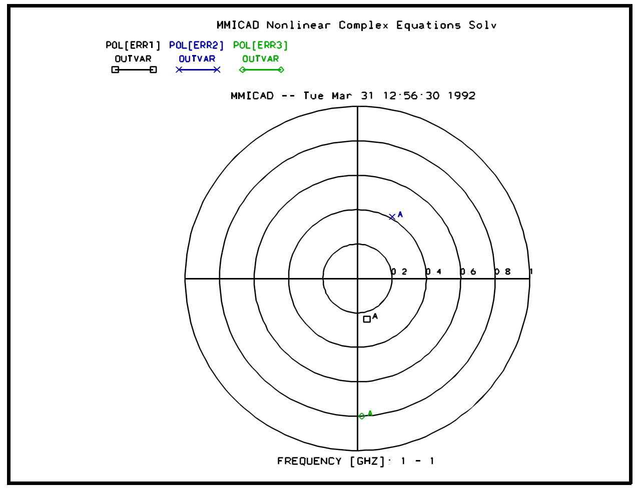

Figure 2 Residues before Optimization

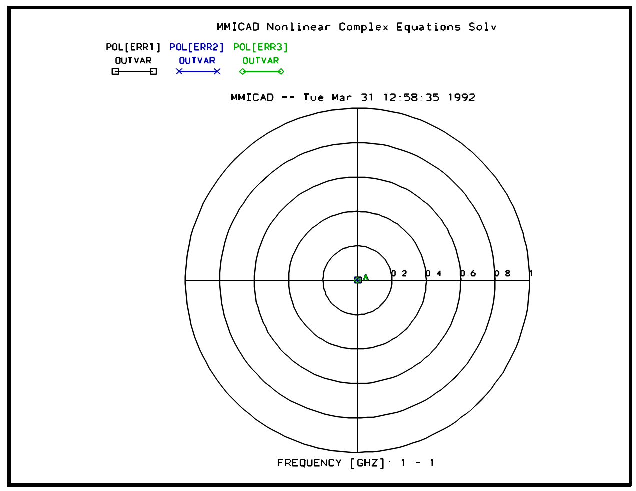

Figure 3 Residues after Optimization