Using MMICAD to Calculate Minimum Noise Measure

by Jon Hagen

Designers of low-noise amplifiers know that the fundamental figure-of-merit for an amplifier or a transistor is noise measure. This parameter is given by M = (F-1)/(1-1/G) where F is the conventional noise figure and G is the available gain. Noise figure alone is a poor figure-of-merit since an amplifier designed to have the best possible noise figure (F approaching unity) will inevitably have no useful gain (G approaching unity).

Noise measure, on the other hand, takes gain into account. It is easily shown, using the formula for the noise figure of a cascade of stages, that M+1 is the noise figure of an infinite cascade of identical amplifiers whose individual noise measure is M. The noise temperature of such a cascade is M x 290 degrees and is known as T-infinity. Normally T-infinity is approached with only two or three stages; the gain becomes high enough that subsequent stages will hardly affect the noise figure.

Note that M depends on the source impedance since both F and G are functions of source impedance. Neither F and G are functions of the output termination but G, of course, contains the S-parameters. For any transistor (or transistor dressed with feedback and/or stabilization elements), the minimum noise measure with respect to source impedance is a key parameter. An amplifier, no matter what circuit topology it employs, cannot have a noise measure better than the minimum noise measure of its best transistor. Lossless feedback, for example, can lower the minimum noise figure but leaves minimum noise measure invariant.

Poole and Paul[1] derived a convenient procedure to calculate Mmin directly from the noise parameters and S-parameters of a given device. Their procedure, also included in a recent textbook[2], is easily incorporated into MMICAD which provides a post-processor block where the user can make further computations on parameters produced by the CAD program itself. Following this reference, the centers and radii of the constant noise measure circles are given by

$$\Gamma_m = \frac{M\,|1+\Gamma_{on}|^2 C_1^* + 4r_n|S_{21}|^2\Gamma_{on}}{M\,|1+\Gamma_{on}|^2 P + |S_{21}|^2(4r_n-W)} \qquad \textbf{(1)}$$

$$\gamma_m = \frac{\sqrt{M^2 M_a + M M_b + M_c}}{M\,|1+\Gamma_{on}|^2 P + |S_{21}|^2(4r_n-W)} \qquad \textbf{(2)}$$

where

$$P = |S_{21}|^2 + |S_{11}|^2 - |\Delta|^2$$

$$Q = |S_{22}|^2 + |S_{21}|^2 - 1$$

$$W = |1+\Gamma_{on}|^2(F_{\min}-1)$$

$$M_a = |1+\Gamma_{on}|^4(PQ + |C_1|^2)$$

$$M_b = |1+\Gamma_{on}|^2|S_{21}|^2(8r_n Re(\Gamma_{on}C_1) - (4r_n|\Gamma_{on}|^2 + W)P - (W-4r_n)Q)$$

$$M_c = |s_{21}|^4(W - 4r_n(1-|\Gamma_{on}|^2))W$$

The value of the minimum noise measure can be found by considering the noise measure circle of zero radius, i.e., set γ_m equal to zero in (2). This results in

$$M_{\min} = \frac{-M_b \pm \sqrt{M_b^2 - 4M_a M_c}}{2M_a} \qquad (3)$$

The value of the minimum noise measure is taken as the smallest nonnegative value of M_{min} given by (3). The source reflection coefficient which results in the minimum noise measure can now be obtained by employing (1)

$$\Gamma_{om} = \frac{M_{\min}|1+\Gamma_{on}|^2 C_1^* + 4r_n|S_{21}|^2\Gamma_{on}}{M_{\min}|1+\Gamma_{on}|^2 P + |S_{21}|^2(4r_n-W)} \qquad (4)$$

The output reflection coefficient of the device, where Γ_{om} is presented to the input port, is given by (2) - (4)

$$\Gamma_{out} = \frac{S_{22} - \Delta\,\Gamma_{om}}{1 - S_{11}\Gamma_{om}}$$

Figure 1 is the listing of a MMICAD file with a block that uses the Poole and Paul procedure to calculate minimum noise measure as well as the associated source reflection coefficient. This file also notes and corrects two errors contained in the original and also reprinted in 2). In this example the noise parameters and S-parameters were not derived from a circuit block preceding the procedure block. Instead, values used by Poole and Paul in their example were fed directly into the procedure block. The output results are shown in Figure 2. The minimum noise measure is 1.006. (Poole and Paul had erroneously calculated 1.303 which is actually greater than the noise measure, 1.290, associated with the source impedance that gives the minimum noise figure). Note that the program also uses the source reflection coefficient corresponding to Mmin to directly calculate the associated noise figure and associated available gain. These are then plugged into the definition of noise measure to verify that the noise measure associated with this source impedance is indeed the minimum noise measure. The straightforward syntax used in the MMICAD example should allow the reader to adapt this procedure directly to his or her own use.

[1]C.R. Poole and D.K. Paul, IEEE Transactions on Microwave Theory and Techniques, Nov. 1985, p. 1254.

[2]G.D. Vendelin, A.M. Pavio, and U. Rohde, Microwave Circuit Design Using Linear and Nonlinear Techniques, John Wiley & Sons, 1990.

FILE NAME: NMEASURE.CKT

REFERENCE: "Optimum Noise Measure Terminations for Microwave Transistor Amplifiers", Poole & Paul, IEEE-MTT, Nov. '85, pg 1254

NOTES: There are two errors in the above reference:
(1) - The first term in the expression for Q should be S22 not S21
(2) - M_c should be multiplied by W

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!CKT                                    !no CKT block is required in this example

VAR                                     !The variable block
S11={0.5029 + J*0.5208}                 !
S22={0.2700 + J*0.5537}                 !
S21={-0.3592 - J*1.2525}                !directly input the example data used in the reference
S12={0.4883 - J*0.5236}                 !
FMIN=1.479                              !
GOPTN={-0.5258 +J*0.3285}               !
RN=12
RNN={RN/50}                             ! normalized source resistance
DELTA={S11*S22-S12*S21}                 ! calculate Mmin as per reference
C1={S11-CNJ(S22)*DELTA}
P={MAG(S21)^2 + MAG(S11)^2 - MAG(DELTA)^2}
Q={MAG(S22)^2 + MAG(S21)^2 -1.0}
W={MAG(1 +GOPTN)^2 * (FMIN -1)}
MA={MAG(1+GOPTN)^4*(P*Q+MAG(C1)^2)}
MB={MAG(1+GOPTN)^2 *MAG(S21)^2 * (8*RNN*RE(GOPTN*C1) - (4*RNN*MAG(GOPTN)^2 +W) * P-(W-4*RNN) *Q)}
MC={MAG(S21)^4*(W-4*RNN*(1-(MAG(GOPTN))^2))*W}
MMIN1={(-MB+SQRT(MB^2-4*MA*MC))/(2*MA)}
MMIN2={(-MB-SQRT(MB^2-4*MA*MC))/(2*MA)}
!The minimum noise measure is MMIN1 or MMIN2 whichever has the highest +ve value
!IF VALUE IS <=0 THEN VALUE=0
MM1|MMIN1=0:0:MMIN1
MM2|MMIN2=0:0:MMIN2
! IF DIFF<=0 THEN NM=MM1 ELSE NM=MM2
DIFF={MM2-MM1}
MINNM|DIFF=MM1:MM1:MM2
! calculate the associated gamma source
GOPTMNUM={MINNM*(MAG(1+GOPTN)^2)*CNJ(C1) +4*RNN*(MAG(S21)^2)*GOPTN}
GOPTMDEN={MINNM*(MAG(1+GOPTN)^2)*P+(MAG(S21)^2)*(4*RNN-W)}
GS={GOPTMNUM/GOPTMDEN}
!needed for the associated available gain
NUMGA={MAG(S21)^2*(1-MAG(GS)^2)}
DENGA={(1-MAG(S22)^2) + MAG(GS)^2 * (MAG(S11)^2 - MAG(DELTA)^2) -2*RE(GS*C1)}
!needed for the associated noise figure
FNUM={4*RNN*MAG(GS-GOPTN)^2}
FDEN={(1-MAG(GS)^2)*MAG(1+GOPTN)^2}

PROC                                    !Generate the outputs to display
GOP=GS                                  !SOURCE REFLECTION COEFFICIENT FOR MINIMUM NOISE MEASURE
GAV=NUMGA/DENGA                         !AVAILABLE GAIN ASSOCIATED WITH GOPT
NOF=FMIN+FNUM/FDEN                      !NOISE FIGURE ASSOCIATED WITH GOPT
CMNM=(NOF-1)/(1-1/GAV)                  !NOISE MEASURE ASSOCIATED WITH GOPT
MNM=MINNM                               !CALCULATED MINIMUM NOISE MEASURE

OUT
OUTVAR MAG[GOP] TABLE T                 !DISPLAY RESULTS IN TABULAR FORMAT
OUTVAR ANG[GOP] TABLE
OUTVAR RE[NOF] TABLE
OUTVAR RE[GAV] TABLE
OUTVAR RE[MNM] TABLE
OUTVAR RE[CMNM] TABLE

FREQ                                    !dummy frequency block
STEP 1
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Figure 1 MMICAD File Listing

Source reflection coefficient for minimum noise measure (GOPTM):	0.560 @ 161.03°
Minimum noise measure (MMIN1):	1.006
Available gain, GA, associated with GOPTM:	2.304
Noise figure, F, associated with GOPTM:	1.569
Noise measure, for verification, calculated directly from F and GA:	1.006

Figure 2 Output Results